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THE EFFECTIVE LENGTH OF A CRACK TERMINATED BY A HOLE

LA LONGUEUR EFFICACE D'UNE FISSURE QUI ABOUTIT À UN TROU

by/par

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National Aeronautical Establishment

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SUMMARY

The influence of a small hole at the tip of a crack on the compliance of a cracked specimen is examined. It is shown that the compliance of the specimen with a hole is the same as the compliance of a specimen with an infinitely sharp but slightly longer crack, the increment of length being proportional to the radius of the hole. In the case of mode III straining (the tearing mode) the equivalent sharp crack extends beyond the edge of the hole a distance equal to the radius of the hole. The results are pertinent to the experimental measurement of stress intensity factors.

SOMMAIRE

On étudie l'influence d'un petit trou à fond d'une fissure sur la raideur d'une éprouvette fissurée. On démontre que la raideur de l'éprouvette est égale à celle d'une éprouvette avec une fissure infiniment aiguë mais un peu plus longue, le prolongement étant proportionnel au rayon du trou. Dans le cas du mode III d'ouverture de la fissure (mode de déchirement) la distance entre la pointe de la fissure aiguë équivalente et le bord du trou est égale au rayon du trou. Les résultats ont rapport à la mesure expérimentale des facteurs d'intensité des contraintes.

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THE EFFECTIVE LENGTH OF A CRACK TERMINATED BY A HOLE

1.0 INTRODUCTION

It is a well-known result of fracture mechanics that the stress intensity factor for a cracked specimen can be related to the change in compliance of the specimen as the crack extends. This result provides the basis for experimental determinations of stress intensity factors. The determination requires measurement of the specimen's compliance for a range of crack lengths and the calculation of rate of change of compliance from the data.

When stress intensity factors are determined in this way, it is a common experimental practice to terminate the crack with a small drilled hole. Use of a hole facilitates measurement of crack lengths by providing a smooth well-defined edge whose location is easily set and measured. There is the further advantage that the hole blunts the crack tip, thus reducing the danger that the crack will extend when the specimen is loaded for the measurement of compliance.

Use of a hole, however, has the disadvantage of altering the geometry of the crack tip and the neighbouring stress field. Since the stress intensity factor is a theoretical concept based on the idealization of an infinitely sharp crack, it is not immediately clear how experimental measurements, obtained from a crack terminated in a hole, should be interpreted. One may reasonably assume that the perturbation introduced by the hole is small provided that the diameter of the hole is sufficiently small, but it is difficult to say anything more precise without further analysis.

The present note offers an analysis of the change in compliance caused by the presence of a hole at the end of a crack. It is found that the compliance of the specimen with a hole is the same as the compliance of a specimen with an infinitely sharp, but somewhat longer, crack. The increment of length between the crack ending in a hole and the equivalent sharp crack is found to be directly proportional to the radius of the hole, with the constant of proportionality being essentially the same fixed quantity for all configurations of specimen.

Although the constant of proportionality is independent of specimen configuration it is not independent of mode of straining. Different constants apply to the three basic modes of straining, that is to the opening mode, the sliding mode and the tearing mode. Determination of the constants requires an analysis of the stress field surrounding the hole. The analysis is carried out in this report for the tearing mode but not, because of mathematical difficulties, for the more important opening and sliding modes. In the case of the tearing mode it is found that the equivalent sharp crack extends beyond the edge of the hole a distance equal to the radius of the hole.

The present results complement previous results of Bowie and Neal (Ref. 1) for shallow edge notches, and supercede a proposal of Srawley, Jones and Gross (Ref. 2) for equivalent crack length which is now seen to be erroneous.

2.0 ANALYSIS

2.1 Comparison of Real and Ideal Specimens

Consider two cracked specimens A and B of general shape, as shown in Figure 1. Specimen A is the ideal specimen in which the crack tip is infinitely sharp, while specimen B in the real specimen in which the crack is terminated by a small hole. Except for the different configuration at the crack tip the two specimens are of identical shape and both are loaded in the same way. The task is to relate the compliance of the ideal specimen, from which the crack-tip stress intensity factor can be determined, to the compliance of the real specimen, which is the quantity that is actually measured experimentally. Since the compliance in this context is essentially a measure of the strain energy in a body, the task is equivalent to obtaining a relation between the strain energies of the real and ideal specimens.

If the stress field σ_{ij} which exists in the ideal specimen A were applied to the real specimen B, it would satisfy the boundary conditions on the external boundary and on the crack but would leave residual tractions T_i^* on the surface of the hole. The residual tractions T_i^*, which are self-equilibrating, can be annulled by superposing a stress field σ'_{ij} which leaves tractions $-T_i^*$ on the surface of the hole and which satisfies zero boundary conditions elsewhere. Thus the stress field σ''_{ij} in the real specimen B is the superposition of fields σ_{ij} and σ'_{ij};

$$\sigma''_{ij} = \sigma_{ij} + \sigma'_{ij} \tag{1}$$

If ϵ_{ij}, ϵ'_{ij}, ϵ''_{ij} are the strains correcponding to stresses σ_{ij}, σ'_{ij}, σ''_{ij} respectively, the strain energy density in the real specimen is

$$\frac{1}{2}\sigma''_{ij}\epsilon''_{ij} = \frac{1}{2}(\sigma_{ij} + \sigma'_{ij})(\epsilon_{ij} + \epsilon'_{ij})$$

$$= \frac{1}{2}\sigma_{ij}\epsilon_{ij} + \frac{1}{2}\sigma'_{ij}\epsilon'_{ij} + \sigma'_{ij}\epsilon_{ij} \tag{2}$$

where we have used the fact $\sigma_{ij}\epsilon'_{ij} = \sigma'_{ij}\epsilon_{ij}$ which is a consequence of the linear relation between stress and strain. The strain energies of the specimens are obtained by integration of the strain energy densities over the regions occupied by the specimens. It is convenient to define two regions R_1 and R_2 relative to either specimen, where R_1 is the region exterior to the boundary of the hole and R_2 is the region within the boundary of the hole. Thus the ideal specimen A occupies the combination of regions R_1 and R_2 while the real specimen B covers region R_1 only. The strain energies U_A and U_B of specimens A and B therefore are

$$U_A = \int_{R_1} \frac{1}{2}\sigma_{ij}\epsilon_{ij}\,dR + \int_{R_2} \frac{1}{2}\sigma_{ij}\epsilon_{ij}\,dR \tag{3}$$

$$U_B = \int_{R_1} \frac{1}{2}\sigma''_{ij}\epsilon''_{ij}\,dR \tag{4}$$

Now integration of (2) over region R_1 and substitution from (3) and (4) yields

$$U_B = U_A - \int_{R_2} \frac{1}{2}\sigma_{ij}\epsilon_{ij}\,dR + \int_{R_1} \frac{1}{2}\sigma'_{ij}\epsilon'_{ij}\,dR + \int_{R_1} \sigma'_{ij}\epsilon_{ij}\,dR \tag{5}$$

The first integral in (5) can be transformed in the usual way into an integral around the boundary of region R_2 of the product of the surface tractions and displacements associated with the stress field σ_{ij}. Since this field leaves residual tractions T_i^* on region R_1, then it leaves equal and opposite tractions $-T_i^*$ on region R_2. Hence

$$\int_{R_2} \frac{1}{2}\sigma_{ij}\epsilon_{ij}\,dR = -\oint_H \frac{1}{2}T_i^* u_i\,ds \tag{6}$$

where u_i is the displacement field associated with σ_{ij} and where H denotes the boundary of the hole. The third integral in (5) can be transformed in the same way into an integral around the boundary of region R_1. Since the tractions associated with the stress field σ'_{ij} are zero everywhere except on the boundary of the hole, where they are equal to $-T_i^*$, we have

$$\int_{R_1} \sigma'_{ij}\epsilon_{ij}\,dR = -\oint_H T_i^* u_i\,ds \tag{7}$$

It follows from (5), (6) and (7) that

$$U_B = U_A + \int_{R_2} \frac{1}{2} \sigma_{ij} \epsilon_{ij} \, dR + \int_{R_1} \frac{1}{2} \sigma'_{ij} \epsilon'_{ij} \, dR \tag{8}$$

Formula (8) gives the relation between the strain energies U_A, and U_B, and it will be useful if the two integrals can be easily evaluated. Note, however, that the analysis so far applies to all three modes of straining.

2.2 Dimensional Form of the Integrals

Evaluation of the first integral in (8) is straightforward. Since the integration extends only over region R_2 which covers the immediate vicinity of the crack-tip, the stress field σ_{ij} in R_2 is given with adequate accuracy by the standard expressions for stresses near a crack tip. The energy density can be calculated from these expressions and the integral evaluated. Dimensionally, the result will be of the form

$$\int_{R_2} \frac{1}{2} \sigma_{ij} \epsilon_{ij} \, dR = f_1 \, K_A^2 \, b/M \tag{9}$$

for a specimen of unit thickness, where K_A is the stress intensity factor for the ideal specimen A, b is the radius of the hole, f_1 is a numerical factor and M is the appropriate elastic modulus. For mode I or mode II straining M is Young's modulus E, while for mode III straining M is the shear modulus G.

Evaluation of the second integral in (8) depends on first finding the stress field σ'_{ij} by means of a stress analysis. It is, however, possible to establish a priori the dimensional form of the result. Since the edge of the hole is in the immediate vicinity of the crack tip, the boundary tractions $-T_i^*$ which determine the stress field can be obtained with adequate accuracy from the standard expressions for stresses near a crack tip. The tractions are therefore proportionto to $K_A/\sqrt{b}$ and hence the stress field σ'_{ij} is also proportional to the same factor. The strain energy density then is proportional to K_A^2/bM. The integral of strain energy density, for a specimen of unit thickness, will be proportional to K_A^2/bM times the square of some characteristic length. Since the stress field σ'_{ij} is no doubt localized around the hole, the hole radius b is an appropriate characteristic length, so that the second integral in (8) will be of the form

$$\int_{R_1} \frac{1}{2} \sigma'_{ij} \epsilon'_{ij} \, dR = f_2 \, K_A^2 \, b/M \tag{10}$$

where f_2 is a numerical factor. Because of the localization of the stress field σ'_{ij} around the hole, the factor f_2 can be expected to be largely independent of the overall shape of the specimen.

The relation (8) between the strain energies of the real and ideal specimens then becomes

$$U_B = U_A + (f_1 + f_2) \, K_A^2 \, b/M \tag{11}$$

2.3 Equivalent Crack Length

The stress intensity factor for the ideal specimen is related to the strain energy release rate by

$$\frac{dU_A}{da} = \frac{K_A^2}{2M} \tag{12}$$

where a denotes the crack length. The measured stress intensity factor K_B, that is the stress intensity factor determined from compliance measurements on the real specimen, is in effect calculated from the analogous expression

$$\frac{dU_B}{da} = \frac{K_B^2}{2M} \tag{13}$$

By differentiating (11) with respect to a, and using (12) and (13) we find

$$K_B^2 = K_A^2 + 4(f_1 + f_2)\, b\, K_A\, (dK_A/da) \tag{14}$$

In performing the differentiation we have assumed, as will be verified, that f_2 is not a function of the crack length a. This assumption is reasonable in view of the expected concentration of the stress field σ_{ij} around the hole. Taking the square root of (14) gives

$$K_B = K_A \left\{1 + 4(f_1 + f_2)\, b\, (dK_A/da)/K_A\right\}^{\frac{1}{2}} \tag{15}$$

The second term in the curly bracket is no doubt a small correction so it should be legitimate to approximate the bracket by the first two terms of its binomial expansion. Thus

$$K_B = K_A \left\{1 + 2(f_1 + f_2)\, b\, (dK_A/da)/K_A\right\} \tag{16}$$

or

$$K_B = K_A + 2(f_1 + f_2)\, b\, (dK_A/da) \tag{17}$$

which is the formula relating the exact stress intensity factor K_A to the experimentally measured factor K_B.

The geometrical interpretation of (17) is shown in Figure 2. It may be seen that if K_A is the stress intensity factor for a crack of length a, then K_B is the stress intensity factor for an ideal crack whose length is greater than a by the amount $2(f_1 + f_2)\, b$. In other words, terminating a crack by a small hole increases its effective length by the amount $2(f_1 + f_2)\, b$.

The next step in the analysis is to determine the values of the factors f_1 and f_2.

2.4 Factors f_1 and f_2 for Mode III Straining

The preceding analysis applies to all three modes of crack straining. It will be recalled that the three basic modes of straining are the crack opening, also known as mode I; the sliding mode also known as mode II; and the tearing mode, or mode III. From now on the analysis will deal with mode III only. Attention is confined to the tearing mode solely because the mathematics involved are simple enough to cope with. The mathematical analysis required for modes I and II is, by comparison, far more difficult and will not be attempted at this time.

Up to now the precise location of the centre of the hole has not been specified. The preceding analysis is valid so long as the tip of the crack lies somewhere within the boundary of the hole. It is now necessary to fix the location of the hole, and for convenience we take the centre of the hole to lie at the point corresponding to the tip of the crack in the ideal specimen. We also introduce a system to coordinates with origin at the centre of the hole and oriented so that the crack is aligned with the negative x-axis as shown in Figure 3.

2.4.1 The Factor f_1

For mode III straining the stresses near the tip of an infinitely sharp crack are (Refs. 3,4)

$$\sigma_{xz} = \frac{-K_{III} \sin(\theta/2)}{\sqrt{2\pi r}} \qquad \sigma_{yz} = \frac{K_{III} \cos(\theta/2)}{\sqrt{2\pi r}} \tag{18}$$

or, in polar coordinates

$$\sigma_{rz} = \frac{K_{III} \sin(\theta/2)}{\sqrt{2\pi r}} \qquad \sigma_{\theta z} = \frac{K_{III} \cos(\theta/2)}{\sqrt{2\pi r}} \tag{19}$$

These expressions may be used for the stresses σ_{ij} in the region R_2. The corresponding strain energy density is

$$\begin{aligned} \frac{1}{2} \sigma_{ij} \epsilon_{ij} &= (\sigma_{rz}^2 + \sigma_{\theta z}^2)/2G \\ &= K_{III}^2/4G\pi r \end{aligned} \tag{20}$$

so that the strain energy in region R_2 is

$$\begin{aligned} \int_{R_2} \frac{1}{2} \sigma_{ij} \epsilon_{ij} \, dR &= \int_0^b \int_{-\pi}^{\pi} (K_{III}^2/4G\pi r) \, r \, d\theta \, dr \\ &= K_{III}^2 \, b/2G \end{aligned} \tag{21}$$

On comparing (21) with (9) and remembering that G replaces M for mode III straining, we find that

$$f_1 = \tfrac{1}{2} \tag{22}$$

2.4.2 The Factor f_2

In order to find f_2 the strain energy of the stress field σ'_{ij} must be calculated. To carry out the calculation exactly for specimens of arbitrary shape would be difficult if not impossible and fortunately is not necessary. Upper and lower bounds on the strain energy can be established by using the theorems of minimum potential energy and minimum complementary energy. The bounds turn out to be close enough to fix the strain energy to an adequate degree of approximation.

Application of the principle of minimum potential energy requires the construction of a suitable approximate but compatible displacement field, while application of the principle of minimum complementary energy requires the construction of an approximate stress field which, however, must satisfy the requirements of equilibrium. Since the stress field σ'_{ij} is concentrated around the hole, suitable approximate fields can be constructed in the following way. With centre at the centre of the hole draw the largest possible circle which lies entirely within the specimen and does not extend beyond the opposite end of the crack. Examples of such circles are shown in Figure 4. Let the radius of the circle be c. It may be assumed c is large relative to the hole radius b. In the region outside the circle of radius c the stress and displacement will be taken to be zero. Thus the required boundary conditions on the exterior of the specimen are satisfied. The stresses and displacements inside the circle will be determined so as to satisfy all the requirements of equilibrium and compatibility as well as the boundary conditions on the surface of the hole and on the faces of the crack. If in addition the stresses satisfy the boundary condition of zero traction on the circle of radius c then we will have obtained a stress field which satisfies the equilibrium conditions throughout the entire specimen. By virtue of the theorem of minimum complementary energy, the strain energy calculated from this equilibrium stress field overestimates the correct strain energy. On the other hand, if the displacement field within the circle is made to satisfy the boundary condition of zero displacement on the circumference then we will have obtained a displacement field which satisfies the requirements of compatibility throughout the entire specimen. By virtue of the theorem of minimum potential energy the strain energy calculated from this displacement field underestimates the correct value.

The strain field associated with mode III straining is of the type known as "anti-plane" strain. Anti-plane strain is an out-of-plane shearing action whose characteristics are that the only non-zero component of displacement is the z-component, and that this component is independent of z. Thus, in the usual notation,

$$u_x = u_y = 0 \qquad u_z = u(x,y) \tag{23}$$

It follows that the stresses are given by

$$\sigma_{xz} = G\,\partial u/\partial x \qquad \sigma_{yz} = G\,\partial u/\partial y$$
$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = 0 \tag{24}$$

The stresses must satisfy the equilibrium equation

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = 0 \tag{25}$$

from which it follows

$$\nabla^2 u = 0 \tag{26}$$

At this point it is convenient to change to polar coordinates because they are more suited to the geometry of the hole. Equation (26) in polar coordinates is

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0 \tag{27}$$

while the formulas for stresses in polar coordinates are

$$\sigma_{rz} = G\,\partial u/\partial r \qquad \sigma_{\theta z} = G\,\partial u/(r\partial\theta) \tag{28}$$

The boundary traction T_z^* on the surface of the hole can be found from (19), from which it follows that the boundary condition for the stress field σ'_{ij} at the surface of the hole is

$$\sigma'_{rz}\,|_{r=b} = -\frac{K_{III}\sin(\theta/2)}{\sqrt{2\pi b}} \tag{29}$$

It is easily verified that

$$u = (A\sqrt{r} + B/\sqrt{r})\sin(\theta/2) \tag{30}$$

is a solution of (27) for any values of the constants A and B, and further this solution leaves zero traction on the faces of the crack. By suitable choice of the constants A and B the solution can also be made to satisfy the boundary condition on the hole and the chosen boundary condition on the circle of radius c. The solution which gives zero traction on the circle of radius c is found to be

$$u = \sqrt{2/\pi r}\,(K_{III}b/G)(1 + r/c)(1 - b/c)^{-1}\sin(\theta/2) \tag{31}$$

while the solution which gives zero displacement on the circle of radius c is found to be

$$u = \sqrt{2/\pi r}\,(K_{III}b/G)(1 - r/c)(1 + b/c)^{-1}\sin(\theta/2) \tag{32}$$

The strain energy calculated from the former solution is

$$\frac{K_{III}^2\, b}{2G}\,\frac{(1 + b/c)}{(1 - b/c)} \tag{33}$$

while the strain energy calculated from the latter solution is

$$\frac{K_{III}^2\, b}{2G}\,\frac{(1 - b/c)}{(1 + b/c)} \tag{34}$$

As already explained, these two values bracket the correct strain energy of the field σ'_{ij}. Since c may be assumed large relative to b, it follows from (33) and (34) that we have approximately

$$\int_{R_1} \frac{1}{2}\, \sigma'_{ij}\, \epsilon'_{ij}\, dR = K_{III}^2\, b/2G \tag{35}$$

to within an error of order b/c or less. On comparing (35) with (10) we conclude that

$$f_2 = \tfrac{1}{2} \tag{36}$$

2.5 Equivalent Crack Length for Mode III Straining

As we have already seen, terminating a crack by a small hole increases its effective length by the amount $2(f_1 + f_2)\, b$. In view of the values obtained for f_1 and f_2 this amount is $2b$ for mode III straining. Since the crack tip in the ideal specimen was taken at the centre of the hole, the tip of the effective crack of the real specimen therefore lies a distance b beyond the edge of the hole, where b is the radius of the hole.

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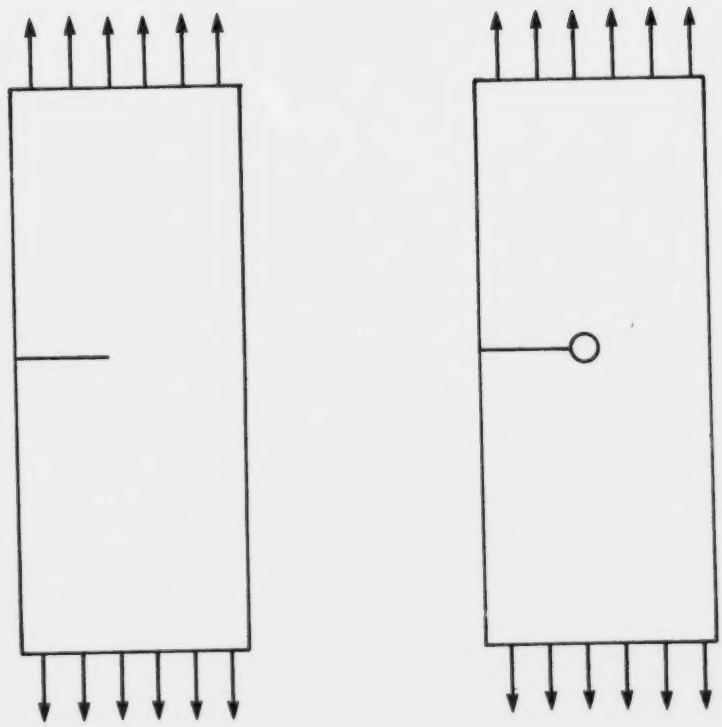

FIG. 1

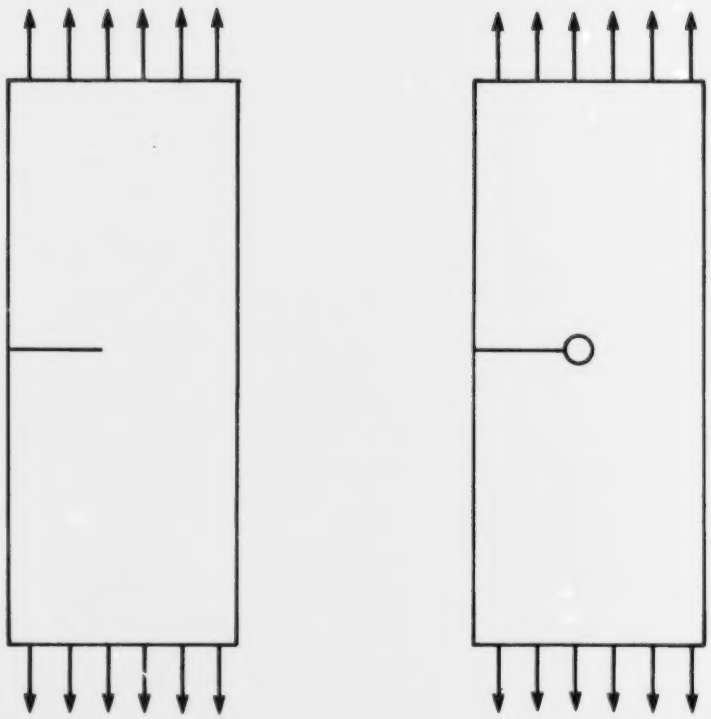

FIG. 1

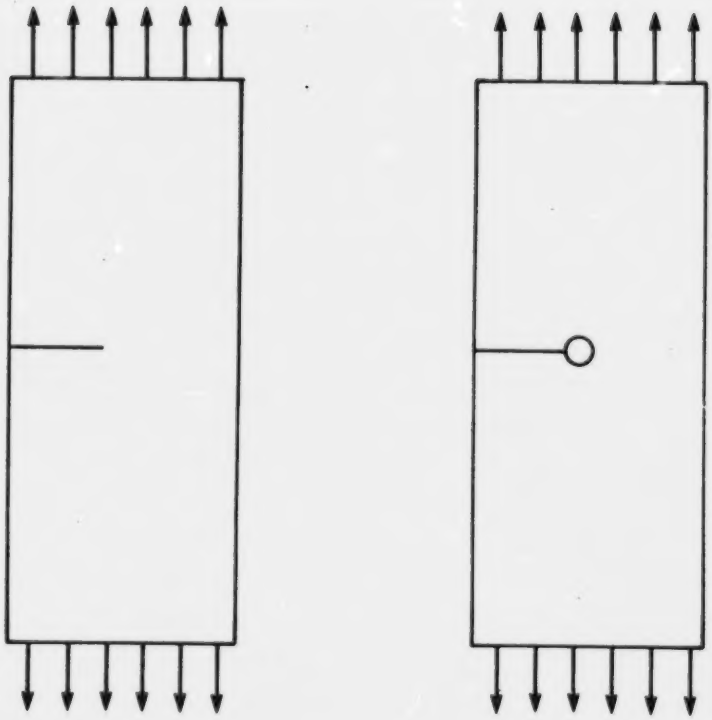

FIG. 1

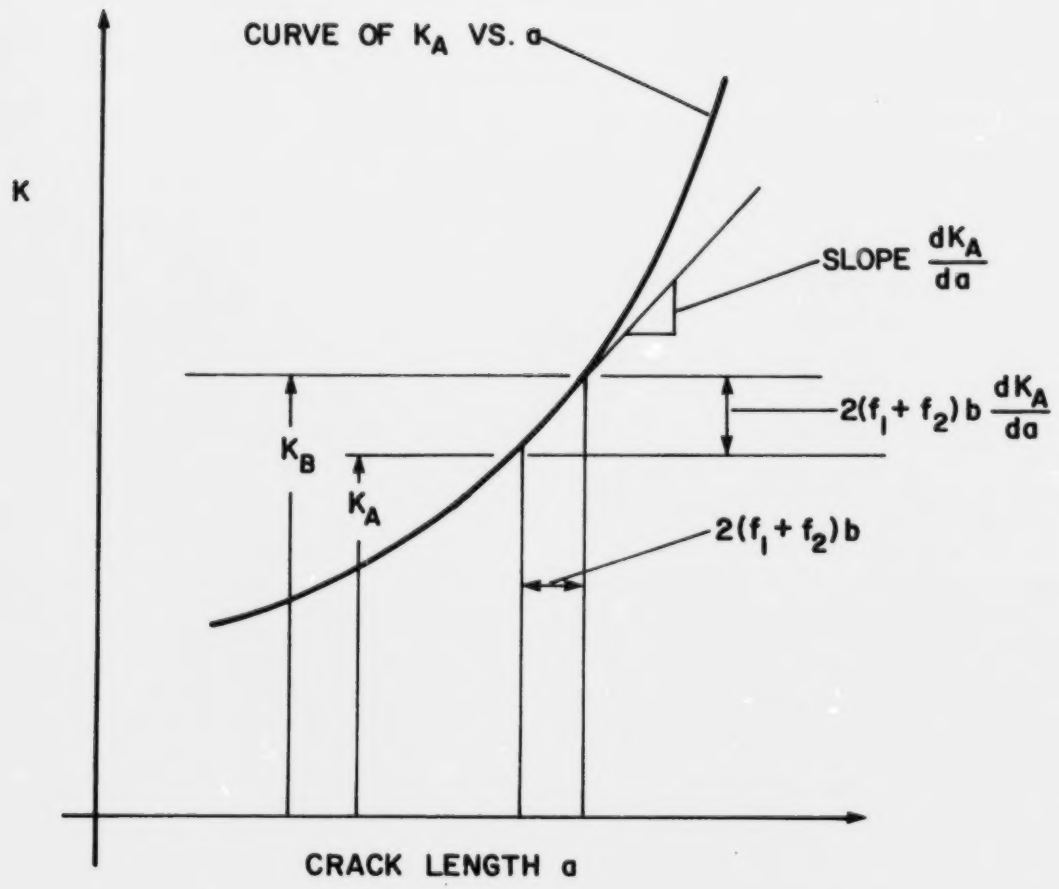
CURVE OF K_A VS. a
K
SLOPE $\frac{dK_A}{da}$
$2(f_1 + f_2)\, b\, \frac{dK_A}{da}$
K_B
K_A
$2(f_1 + f_2)\, b$
CRACK LENGTH a

FIG. 2

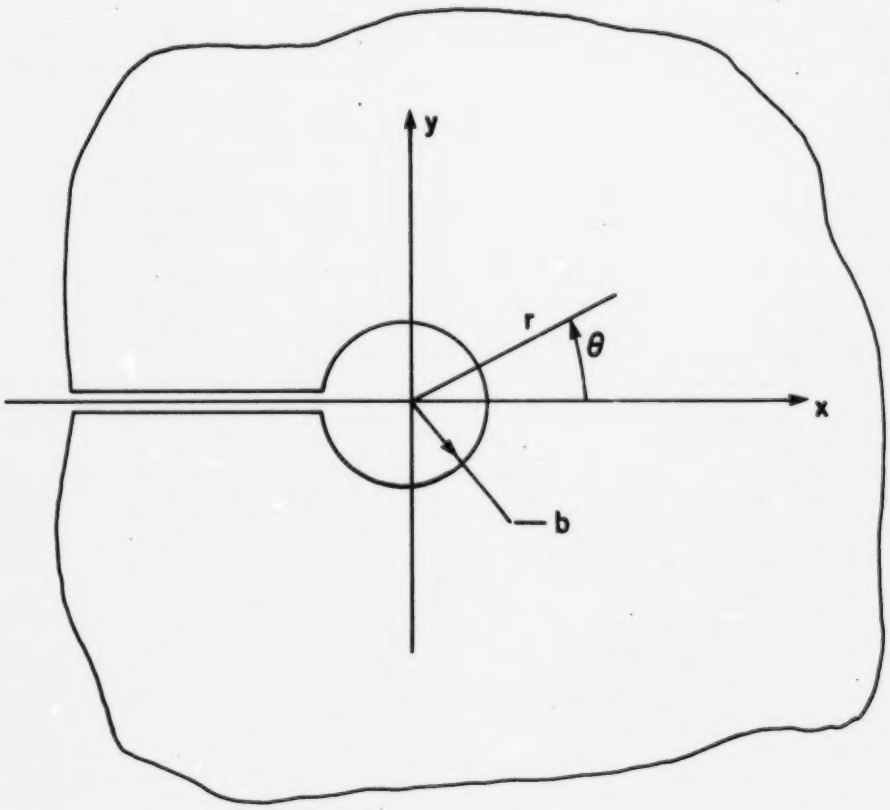

FIG. 3

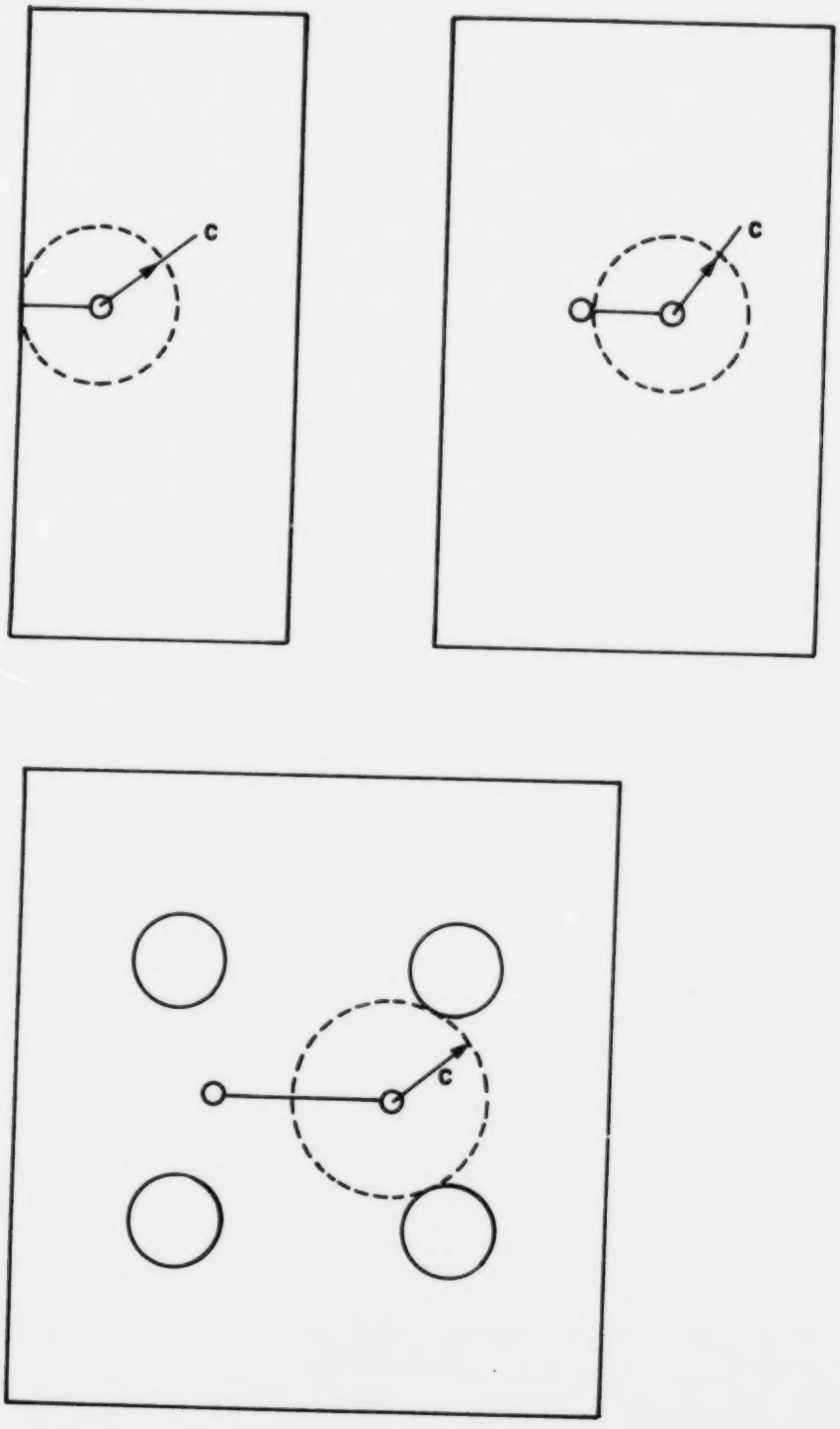

FIG. 4

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1. Cracks — Propagation

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The influence of a small hole at the tip of a crack on the compliance of a cracked specimen is examined. It is shown that the compliance of the specimen with a hole is the same as the compliance of a specimen with an infinitely sharp but slightly longer crack, the increment of length being proportional to the radius of the hole. In the case of mode III straining (the tearing mode) the equivalent sharp crack extends beyond the edge of the hole a distance equal to the radius of the hole. The results are pertinent to the experimental measurement of stress intensity factors.